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Total Pages : 4

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DISCRETE MATHEMATICS–212

(Semester–III)

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all. Select *two* questions each from Section A and B. Question No. IX of Section C is compulsory.

SECTION–A

I. (a) For any sets A and B, prove that

$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

(b) In a class of 60 boys, there are 45 boys who play cards and 30 boys play carom.

Find

(i) How many boys play both games ?

(ii) How many boys play cards only ?

(iii) How many boys play carom only ? (6,9)

II. (a) Give direct proof of

$p \to (q \to s), \sim r \vee p, q \Rightarrow r \to s$.

(b) Show that $(\sim p \wedge \sim q) \to (p \to q)$ is a tautology.

(9,6)

III. Prove with the help of mathematical induction that

$$a + ar + ar^2 + \dots\dots\dots + ar^{n+1} = \frac{a(1-r^n)}{1-r}, \ r \neq 0. \qquad (15)$$

IV. (a) Suppose that R is an equivalence relation on a set X. Then

* $a \in [a], \forall a \in X$.

** $a \in [b]$ if and only if $[a] = [b], \forall a, b \in X$.

*** $[a] = [b]$ or $[a] \cap [b] = \phi, \forall a, b \in X$.

(b) Let R be a relation on a set A = {1, 2, 3, 4} defined by R = {(1, 2), (2, 3), (3, 4), (2, 1)}. Find transitive closure of R. (9,6)

SECTION–B

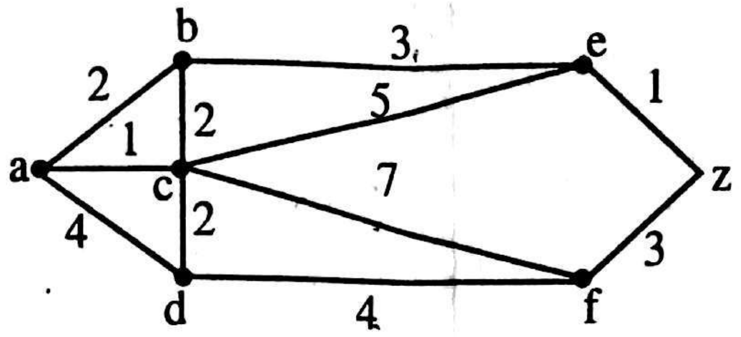
V. (a) Show that $x^4 + 9x^3 + 4x + 7$ is $O(x^4)$.

(b) Prove that if A is a set then identity function I on A is one-one and onto. (7,8)

VI. Let $f : X \to Y$ and $g : Y \to Z$ and let f, g be one-one onto, then prove that $gof\ X \to Z$ is also one-one and onto. Also $(gof)^{-1} = f^{-1}og^{-1}$. (15)

VII. (a) Prove that the sum of the degrees of all the vertices in a graph G is equal to twice the number of edges in G.

(b) Find the shortest path between a and z.



(7,8)

VIII. (a) Prove that a tree with n vertices has $(n - 1)$ edges.

(b) What is the value of post-fix expression

$7\ 2\ 3\ *\ -\ 4\ \uparrow\ 9\ 3\ /\ +$ (8,7)

SECTION–C

Compulsory Question

IX. (a) Let A and B are two sets. Prove that $A - B = A \cap B^C$. (2)

(b) Prove that $p \rightarrow q \equiv (\sim p) \wedge q$. (1)

(c) Let R be a relation on a set A = {1, 2, 3} defined by R = {(1, 1), (1, 2), (2, 3)}. Find the reflexive and symmetric closure of R. (2)

(d) If $f, g : R \rightarrow R$ are defined respectively by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$. Find fog and gof. (2)

(e) If A = $\{a, b, c, d, e\}$, S = $\{a, c, e\}$. Find C_S and $C_\varnothing$.

(f) Is there a graph with 8 vertices of degree 2, 2, 3, 6, 5, 7, 8, 4 ? Justify your answer. (2)

(g) Consider the completely parenthesized algebraic expression $(a - b) \times (c + (d \div e))$. Find its preorder, postorder and inorder search. (2)

(h) Find K, if a K–regular graph with 8 vertices has 12 edges. Also draw K–regular graph. (2)